# Graphing Rational Functions - Notes B1

Wednesday, September 11, 2019 1:43 PM

**STEPS:**

① Factor & simplify the function
- always rewrite the simplified function.

② If you were able to cancel out a factor you HAVE A HOLE in the graph
- Hole is a removable discontinuity
  - A hole is a point $(x, y)$

How to find it:
a.) Set cancelled out factor = 0
b.) Solve for x
c) Take that value & plug in to simplified function & get a y-value.
d.) $(x, y)$ is the hole.

③ VERTICAL ASYMPTOTE(S)
- these are non-removable discontinuities
- VA's happen when you have variables in the denominator that do not cancel out.

How to find them:
a.) Set the remaining factors (in the bottom of the simplified function) = 0
b.) Solve for x
c.) The VA is a vertical line @ x = #

④ HORIZONTAL ASYMPTOTES
3 cases

| DEGREE | HA |
|---|---|
| ① Denominator Degree is larger than numerator | $y = 0$ |
| ② Numerator & Denominator Degrees are the same | $y =$ RATIO of LEADING COEFFICIENTS |
| ③ Numerator Degree is larger than Denominator | NO HA (could be SA) |

you can't have an HA and a SA

⑤ SLANT ASYMPTOTES
- These occur when numerator is larger than denominator degree by EXACTLY ONE

How to find them:
a.) Use synthetic division
b.) Divide by the VA (if no VA, no SA)
c.) Dividend (#'s up top) are the coefficients of the simplified numerator (standard form)
d.) Disregard remainder → answer is from 2 #'s underneath the synthetic division
e.) write answer as $y = mx + b$

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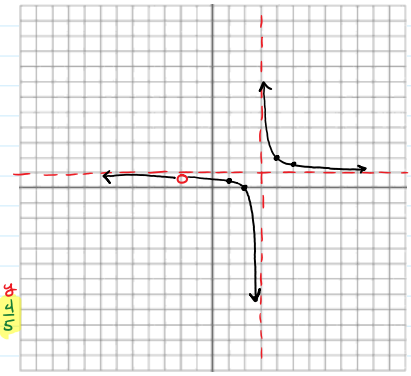
① $y = \dfrac{x^2 - 4}{x^2 - x - 6} = \dfrac{(x+2)(x-2)}{(x+2)(x-3)}$

$y = \dfrac{x-2}{x-3}$



Hole: $(-2, 4/5)$ $x + 2 = 0$, $x = -2$

$y = \dfrac{x-2}{x-3} = \dfrac{-2-2}{-2-3} = \dfrac{-4}{-5} = \dfrac{4}{5}$

VA: $x = 3$ $x - 3 = 0$, $x = 3$

HA: $y = 1$ exp. are tied. Both Leading Coeff's are 1. Ratio: $\frac{1}{1} = 1$

SA: none

Domain: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

| x | y |
|---|---|
| 1 | 1/2 |
| 2 | 0 |
| 3 | VA |
| 4 | 2 |
| 5 | 3/2 |

② $y = \dfrac{2x^2 + x - 1}{x - 1}$

$\dfrac{(2x-1)(x+1)}{x-1}$ → VA

Hole: none

VA: $x = 1$ $x - 1 = 0$, $x = 1$

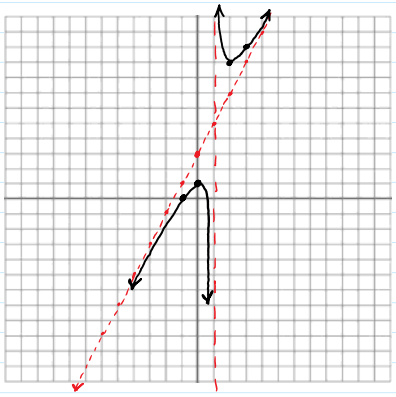
HA: none → CASE 3

SA: $y = 2x + 3$

D: $(-\infty, 1) \cup (1, \infty)$

SA: $1 \mid 2 \quad 1 \quad +1$
$\quad\quad\downarrow \quad 2 \quad 3$
$\quad\quad 2 \quad 3 \quad 2$ → Remainder
$\quad\quad x$ + Constant

$y = 2x + 3$



| x | y |
|---|---|
| −1 | 0 |
| 0 | 1 |
| 1 | VA |
| 2 | 9 |
| 3 | 10 |